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Short Communication

Experimental verification of the MacCormack numerical scheme

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One-dimensional and two-dimensional dam-break flow experiments, that had been performed on two original laboratory installations, were numerically simulated using the MacCormack explicit computational scheme. Accuracy and conservation properties were analysed.

Key words: unsteady flow, dam-break experiments, MacCormack scheme.

1 INTRODUCTION

The MacCormack explicit numerical scheme has gained considerable popularity over the last few years in dealing with one-, and two-dimensional unsteady open channel flows. Reliable results and relative ease by which 'weak solutions' can be obtained, make this scheme particularly suitable for dam-break problems. The performance of the scheme in dealing with one-dimensional dam-break flows has been evaluated by several authors.[1–4] However, there are only few experimental data on two-dimensional flows in literature.[3–5] Therefore, the purpose of this paper is to present some experiences with the application of the MacCormack scheme, especially those referring to two-dimensional dam-break laboratory experiments.

2 GOVERNING EQUATIONS

The unsteady, two-dimensional open channel flow (in the horizontal plane) can be described by the depth-averaged equations of mass and momentum conservation written in the matrix form:

$$\frac{\partial \mathbf{V}}{\partial t}+\frac{\partial \mathbf{G}}{\partial x}+\frac{\partial \mathbf{H}}{\partial y}=\mathbf{T} \quad (1)$$

$$\mathbf{V}=\begin{bmatrix} h \\ uh \\ vh \end{bmatrix} \mathbf{G}=\begin{bmatrix} uh \\ u^2h+\frac{1}{2}gh^2 \\ uvh \end{bmatrix} \mathbf{H}=\begin{bmatrix} vh \\ uvh \\ v^2h+\frac{1}{2}gh^2 \end{bmatrix} \quad (2)$$

$$\mathbf{T}=\begin{bmatrix} 0 \\ gh(S_{ox}-S_{fx}) \\ gh(S_{oy}-S_{fy}) \end{bmatrix} \quad (3)$$

In these equations, x and y are space coordinates and t is the time coordinate. Water depth is designated by h, the depth averaged velocities in x and y directions by u and v respectively, and the acceleration due to gravity by g. Bottom bed slopes are S_{ox} and S_{oy}, while S_{fx} and S_{fy} correspond to the energy gradients that are approximated by the Manning's formula:

$$S_{fx}=\frac{n^2u\sqrt{u^2+v^2}}{h^{4/3}};\ S_{fy}=\frac{n^2v\sqrt{u^2+v^2}}{h^{4/3}} \quad (4)$$

The set of equations, given in the form of the matrix eqn (1) together with the expressions (2) and (3), is a system of quasi-linear hyperbolic partial differential equations. Since the equations are written in the conservation form, they can admit 'weak solutions', i.e. solutions that may lead to discontinuities such as shock waves induced by a dam failure. The MacCormack numerical scheme is one of the several techniques to obtain such solutions. Although the derivatives in eqn (1) are discretized to the first order of accuracy, it is claimed that the scheme is of the second-order of accuracy in both space and time.[6–8] It is also claimed that it does not require shock-fitting procedure[2] — a feature particularly suitable for solving dam-break problems in natural channels.

The MacCormack explicit finite-difference scheme consists of a two step predictor–corrector sequence:[7]

— Predictor step:

$$V^p_{i,j} = V_{i,j} - \frac{\Delta t}{\Delta x}\mathscr{B}_x G_{i,j} - \frac{\Delta t}{\Delta y}\mathscr{B}_y H_{i,j} + \Delta t T_{i,j} \qquad (5)$$

— Corrector step:

$$V^c_{i,j} = V^p_{i,j} - \frac{\Delta t}{\Delta x}\mathscr{F}_x G^p_{i,j} - \frac{\Delta t}{\Delta y}\mathscr{F}_y H^p_{i,j} + \Delta t T^p_{i,j} \qquad (6)$$

— New values:

$$V'_{i,j} = \frac{1}{2}(V_{i,j} + V^c_{i,j}) \; (1 \le i \le N;\; 1 \le j \le M) \qquad (7)$$

In the proceeding equations, $V_{i,j}$ is the vector of the dependent variables from the previous time level, $V^p_{i,j}$ and $V^c_{i,j}$ are the corresponding vectors calculated in the predictor and corrector steps respectively, while $V'_{i,j}$ is the solution vector at the current computational time level. The unknown depth-averaged velocities are finally calculated from $V'_{i,j}$: $u'_{i,j} = (uh)'/h'$ and $v'_{i,j} = (vh)'/h'$.

The spacial difference operators ($\mathscr{B}$ and $\mathscr{F}$) in eqns (5) and (6) refer to backward and forward differencings:

$$\mathscr{B}_x G_{i,j} = G_{i,j} - G_{i-1,j} \text{ and } \mathscr{F}_x G_{i,j} = G_{i+1,j} - G_{i,j} \qquad (8)$$

where the subscript indicates the direction of differencing.

The difference operators are repeated every fourth time step (Table 1), in order to include different flow regimes that might simultaneously occur in the channel.[2,7]

The stability criterion is defined by the Courant–Friedrichs–Lewy condition:[2]

$$\Delta t \le \min\left(\frac{\Delta x}{(u + \sqrt{gh})_{\max}}, \frac{\Delta y}{(v + \sqrt{gh})_{\max}}\right) \qquad (9)$$

3 VERIFICATION OF THE MACCORMACK SCHEME

The experimental verification of the aforedescribed numerical scheme was performed using the measurement results that had been registered on the two experimental rigs in the Hydraulic Laboratory at the Faculty of Civil Engineering in Belgrade.

Table 1. Difference operator sequence

Time step	Predictor x	Predictor y	Corrector x	Corrector y
1	$\mathscr{B}$	$\mathscr{B}$	$\mathscr{F}$	$\mathscr{F}$
2	$\mathscr{F}$	$\mathscr{F}$	$\mathscr{B}$	$\mathscr{B}$
3	$\mathscr{B}$	$\mathscr{F}$	$\mathscr{F}$	$\mathscr{B}$
4	$\mathscr{F}$	$\mathscr{B}$	$\mathscr{B}$	$\mathscr{F}$

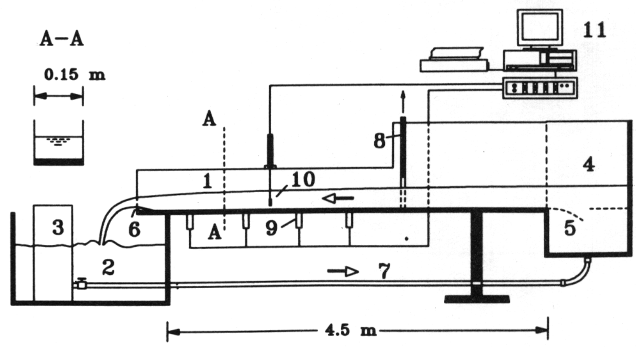

Fig. 1. Experimental installation for investigation of one-dimensional dam-break flows:[12] (1) glass-walled flume, (2) lower tank, (3) pump, (4) upper tank (reservoir), (5) stilling elements, (6) control weir, (7) fluid supply rubber tube, (8) removable gate, (9) depth measurement probes, (10) velocity measurement probes and (11) data acquisition and processing system.

One-dimensional flow experiments

The results from a rig, consisting of 4·5 m long and 0·15 m wide laboratory flume with glass walls and adjustable bottom slope (Fig. 1), were used to verify the numerical simulation of one-dimensional unsteady flow. The flow was caused by a sudden release of the water stored in the reservoir that had been formed in the upstream part of the flume. The membrane 'Druck' probes, together with an electronic data acquisition and processing system were used for water depth measurement at four control profiles situated at $x = 0{\cdot}75$, 1·25, 1·75, 2·25 m from the gate. The glass walls had been covered with a square net ($\Delta x = \Delta z = 1$ cm) and the water levels were recorded by a high-speed camera. Thus it was possible to register the water depths on every 5 cm along the flume.

The equivalent Manning's roughness for the canal was calibrated using the steady flow data.

The evolution of the measured and calculated flow depth profiles of an experiment in which the initial water depth in the reservoir was $H = 0{\cdot}3$ m, the bottom canal slope 0·1% and Manning's coefficient $n = 0{\cdot}009\,\text{m}^{-1/3}\,\text{s}$ are shown in Fig. 3. The upstream boundary condition was defined in accordance with the gate maneuver in this experiment (Fig. 2). The upstream flow discharge was calculated from the equations for the shock movement.[9] No artificial viscosity was used.

For the sake of easier comparison with the measurements, the time step, used in this simulation, was kept constant $\Delta t = 0{\cdot}01$ s, which resulted in the maximum Courant's number value of 0·6.

It can be noticed (Fig. 3) that the calculated flow depth profile follows the shape of the experimental one. The quantitative analysis has shown that the water

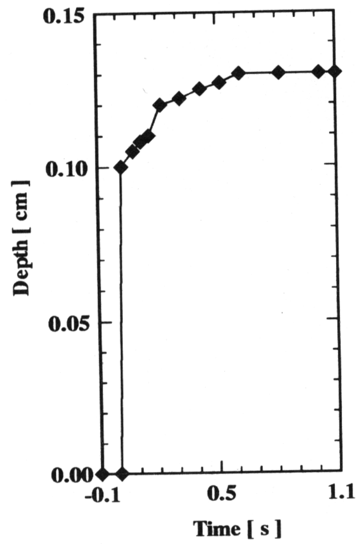

Fig. 2. The stage hydrograph at the gate cross-section, used as the upstream boundary condition.

depth differences are 12% on the average. The wave celerity prediction is good, while the computed wave front height is somewhat higher than the measured one.

The conservation properties' analysis of the MacCormack computational scheme has shown a maximum volume loss/gain of 0·4% for this particular test case. Good conservation properties of the scheme have also been reported in scientific papers for one-dimensional flows (see for instance Refs 1 and 4).

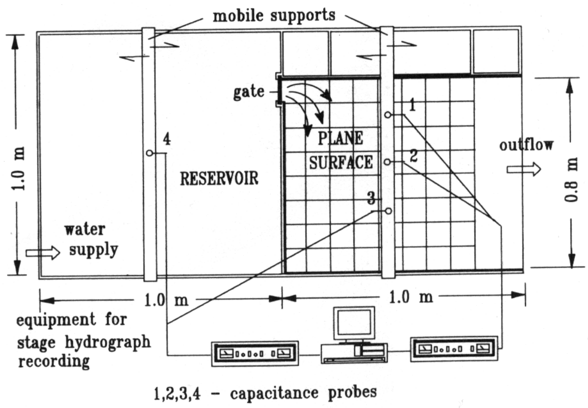

Fig. 4. Pilot laboratory model for the two-dimensional dam-break flow investigation.[13]

Two-dimensional flow experiments

The pilot laboratory installation for the two-dimensional dam-break flow investigation is shown in Fig. 4.

In the physical experiment, the water was released from the reservoir through an opening 0·1 m wide, which was formed by instantaneous removal of a gate between the reservoir and the horizontal surface downstream. The depths' variations at chosen points were recorded by 2-mm thick capacitance probes, while the stream pattern has been filmed by a high speed camera. The capacitance probes had been calibrated in the flume under the steady supercritical flow conditions for the

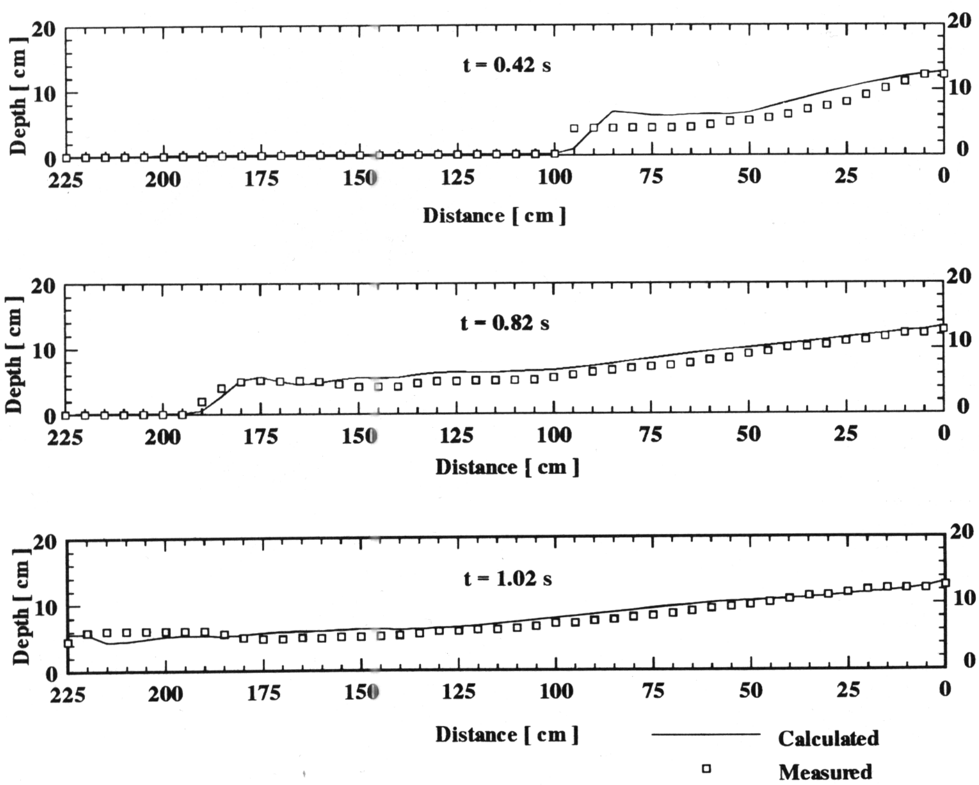

Fig. 3. Comparison of the calculated and the measured flow profiles in the laboratory flume.

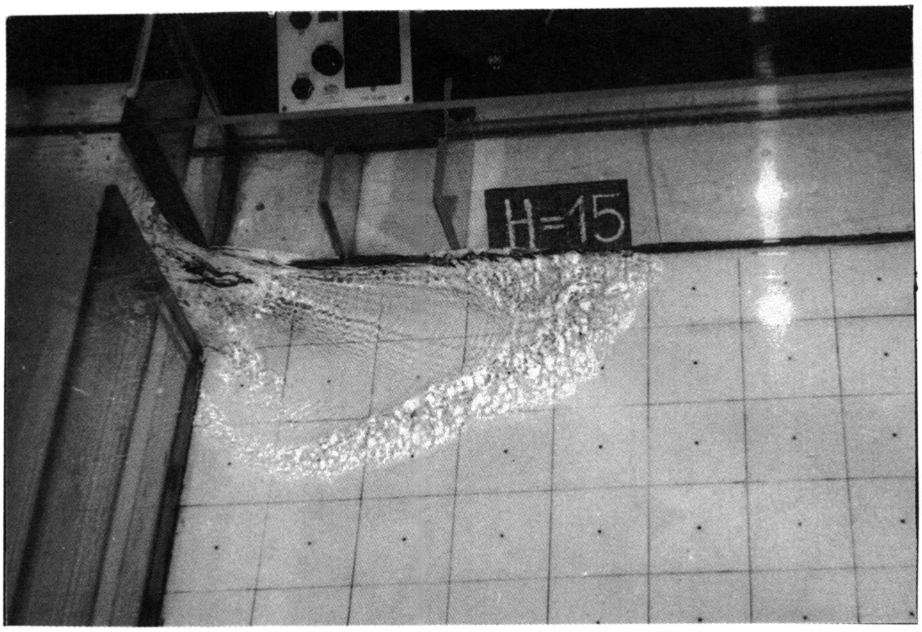

Fig. 5. Unsteady flow experiment on the two-dimensional pilot model at $t = 0{\cdot}6$ s.

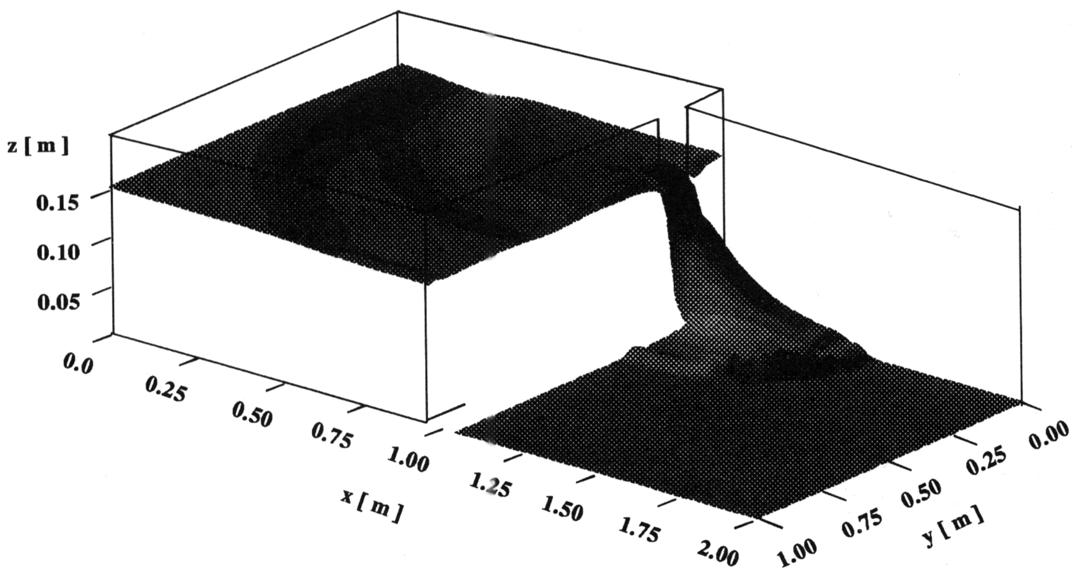

Fig. 6. Numerical simulation of the experiment.

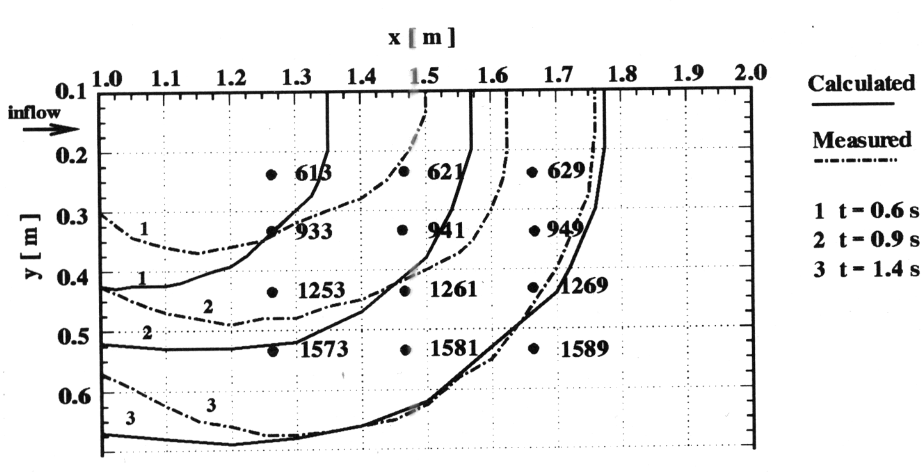

Fig. 7. Wave front advancing.

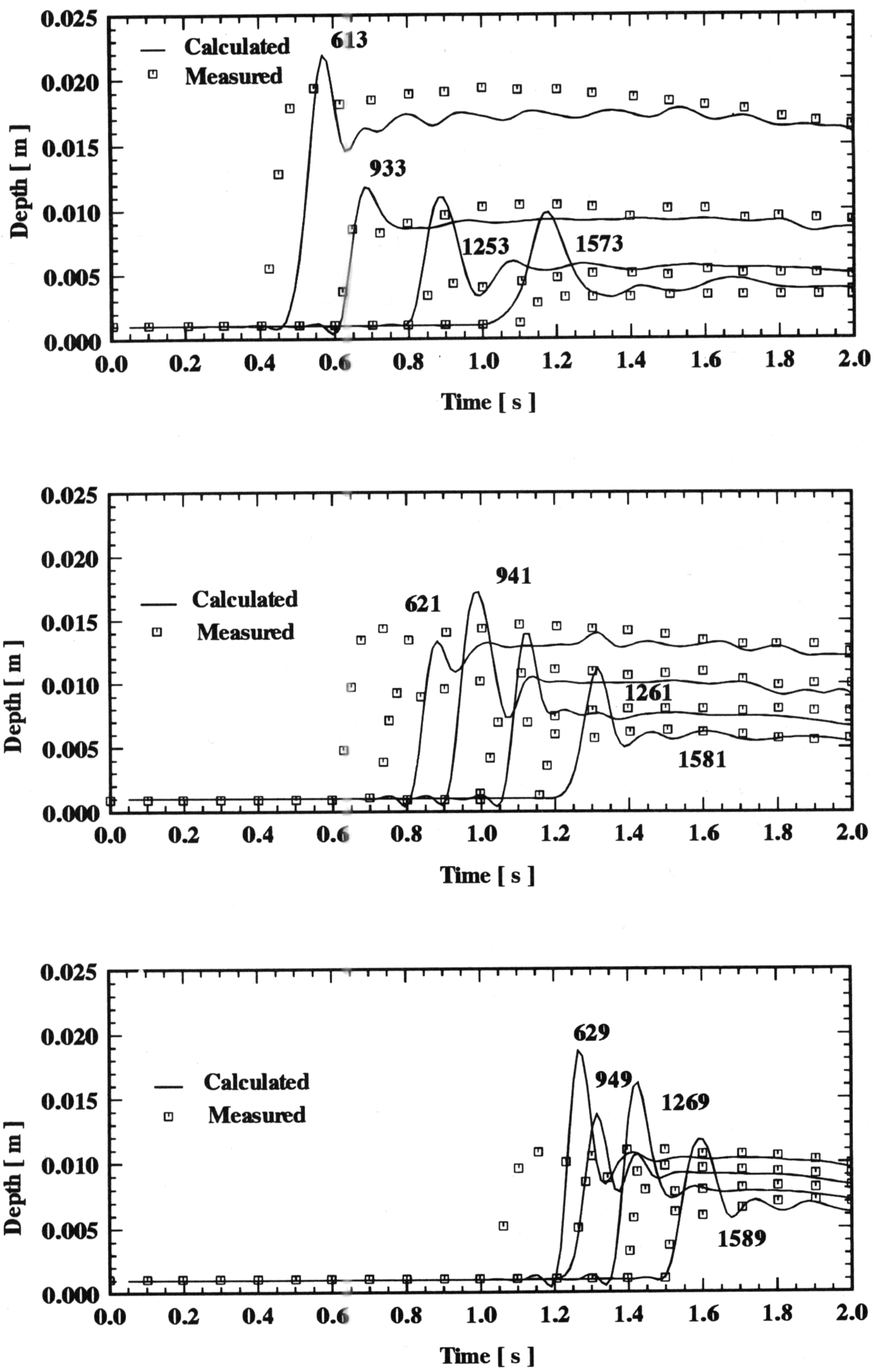

Fig. 8. Calculated and measured stage hydrographs.

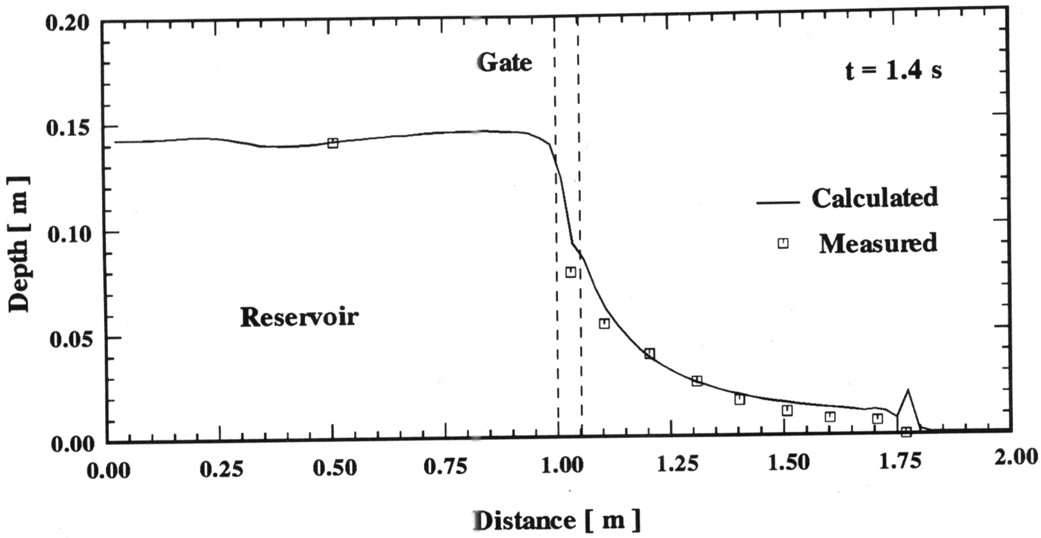

Fig. 9. Calculated and measured water depth longitudinal profile at $t = 1{\cdot}4$.

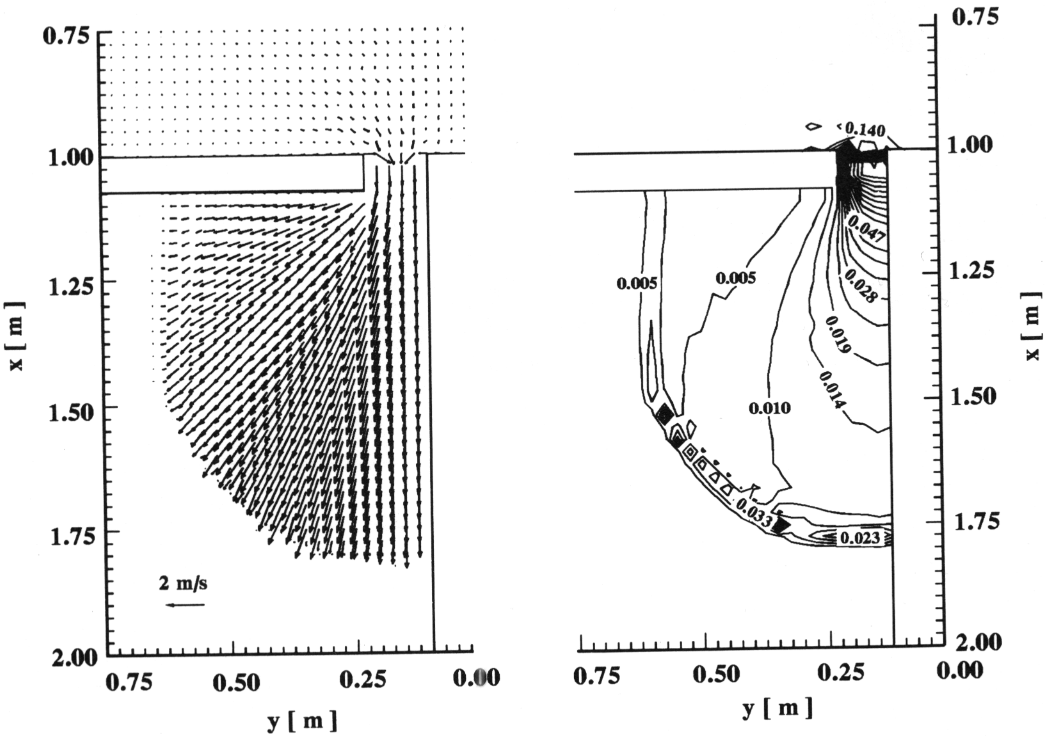

Fig. 10. Vector field and contour lines at $t = 1{\cdot}4$ s.

water 'piling up' effect on the upstream side of the probes, caused by the high wave front velocities.

An experiment in which the initial reservoir water depth was $H = 0{\cdot}15$ m, and the bottom roughness $n = 0{\cdot}01\ \mathrm{m}^{-1/3}$ s is described in this paper. The experiment was numerically simulated using a computational grid with 3360 points, and the individual mesh size of $\Delta x = \Delta y = 0{\cdot}025$ m. The boundary conditions were set in the form of zero velocities u and v at the side walls. The initial condition at the opening was defined according to the theoretical solution for the one-dimensional dam-break problem: $h = (4/9)H$, $u = (8/27)\sqrt{gH}$, and $v = 0$, while for numerical reasons, the initial depth downstream from the reservoir was set to $0{\cdot}001$ m.

Figure 5 depicts the water wave propagation 0·6 s after the gate removal, and Fig. 6 the results of the numerical simulation. It can be noticed that the wave front form has been generally well reproduced using the MacCormack numerical scheme.

The form and the position of calculated and measured wave fronts are presented in Fig. 7. In the short period of time after the flow initiation, the form and the position of the calculated wave front do not coincide with the measured ones ($t = 0{\cdot}6$ and $0{\cdot}9$ s). The recorded wave front is more elongated in the longitudinal direction than the predicted one. This can be explained by applying particular initial conditions at the gate cross-section. Later on, these differences tend to diminish ($t = 1{\cdot}4$ s). The results of some other authors, obtained by different numerical schemes, show similar tendency.[3] A time needed for the wave front to reach the outflow boundary of the physical model is 1·6 s.

The comparison between calculated and recorded stage hydrographs is given in Fig. 8. These hydrographs pertain to grid points 613, 933, ..., 1589, marked in Fig. 7. The water depth differences are the greatest at the onset of hydrographs, due to the dispersive errors of the scheme. No attempt has been made to smooth oscillations by using the artificial viscosity. Apart from these errors, there is a fairly good agreement of the calculated and measured depths (under 18%). A longitudinal water depth profile along the center-line of the opening is presented in Fig. 9 at $t = 1{\cdot}4$ s. The agreement between computed and measured results is satisfactory (under 13%). Dispersion error of the scheme is noticeable at the steep front. More realistic results in the vicinity of the wave front could be obtained (if required) by using the artificial viscosity smoothing technique.[7]

Figure 10 presents the calculated velocity field and the contour lines for the same time instant.

4 CONCLUSIONS

The qualitative and quantitative analysis of the numerical simulation results for one- and two-dimensional laboratory experiments have shown that the MacCormack numerical scheme yields physically plausible results, and can be efficiently used in solving dam-break problems for which a simulation with limited numerical diffusion is required.

The wave celerity and the water depth predictions, made by this computational scheme, for one-dimensional dam-break flows, agree well with the results of the laboratory measurements, as it was reported by other investigators.[1]

The wave front advancing depends, during the initial phase of the two-dimensional numerical simulation, on the imposed initial condition at the dam cross-section. However, the influence of the applied initial condition rapidly decreases in time. The water level differences between the computed and the measured values are within acceptable limits. Additional validation of the two-dimensional model against field data would be valuable.

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